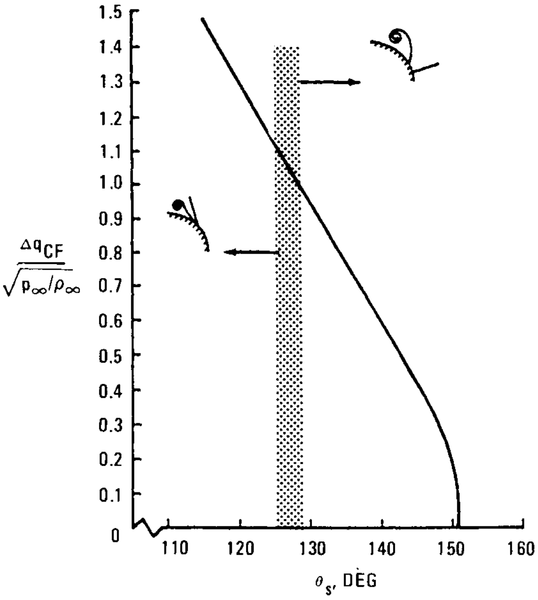


Fig. 5 Vorticity shed into the flowfield as a function of separation point location.

data of Ref. 5 indicate primary separation at 120 deg and secondary separation at 160 deg for this case. The shock configuration transition from an oblique crossflow shock to a detached normal crossflow shock occurs at about $\theta_s \approx 128$ deg (indicated by the shaded area in Fig. 5). It should be pointed out that the jump in velocity at the separation point in the oblique shock cases was computed by subtracting the oblique shock velocity jump from the numerical results. Thus, the jumps in velocity in Fig. 5 represent the jumps across the vortex sheet at separation. The figure shows that this velocity jump goes to zero smoothly as the separation point location due to shock vorticity alone is approached ($\theta_s = 151.3$ deg).

Conclusion

The results of this analysis indicate that there is a relationship between separation produced by shock vorticity and shed vorticity and that both sources of vorticity may be important. The relationship is inferred because the jump in velocity (i.e., vorticity shed) smoothly goes to zero as the shock vorticity separation point location is approached. In fact, it would seem that separation due to shock vorticity alone can be considered a limiting solution of the set of solutions in which vorticity is shed from the surface. In this limiting solution the vorticity shed is zero.

Acknowledgment

This work was supported by the Air Force Office of Scientific Research under Contract F49620-84-C0056.

References

[1]Marconi, F., "Supersonic Conical Separation Due to Shock Vorticity," *AIAA Journal*, Vol. 22, Aug. 1984, pp. 1048–1055.

[2]Salas, M. D., "Recent Developments in Transonic Euler Flow Over a Circular Cylinder," NASA TM 83282, 1982.

[3]Smith, J. H. B., "Remarks on the Structure of Conical Flow," *Progress in Aeronautical Sciences*, Vol. 12, June 1972, pp. 241–271.

[4]Marconi, F., "The Spiral Singularity in the Supersonic Inviscid Flow Over a Cone," AIAA Paper 83-1665, 1983.

[5]Rainbird, W. J., "The External Flow Field About Yawed Circular Cones," AGARD CP30, 1968, pp. 19-1–19-18.

Supersonic Laminar Flow Development in a Square Duct

D. O. Davis* and F. B. Gessner†
University of Washington, Seattle, Washington
and
G. D. Kerlick‡
Informatics General Corporation
NASA Ames Research Center
Moffett Field, California

Introduction

THE nature of laminar flow along a streamwise corner has been previously investigated, primarily for the limiting case of zero-pressure gradient flow along two semi-infinite plates that intersect at 90 deg. The problem is typically analyzed by means of matched asymptotic expansions applied to reduced forms of the conservation equations in order to determine far-field conditions, which then serve as boundary conditions when the conservation equations are applied to the corner layer. The results for both incompressible[1-3] and compressible[4,5] flow show that a crossflow exists in the far field when it is directed toward the corner in the viscous layer immediately adjacent to each plate. This crossflow increases in magnitude as the Mach number increases from subsonic to supersonic values, and is larger in magnitude when the walls are adiabatic, rather than highly cooled.[4,5]

Associated with this behavior is an outwardly directed crossflow along the corner bisector that is not part of a closed vortical pattern.[1-6] Whether or not this crossflow leads to an outward bulging of isovel (axial velocity) contours in the cross plane or, alternatively, total pressure contours, has been the subject of recent controversy. The results of the earlier analyses by Refs. 1–4 all show that isovel contours are undistorted by an outwardly directed crossflow along the corner bisector. A more recent analysis by Nomura[7] has shown, however, that these contours will always bulge outward if the plate intersection angle is between 0 and 180 deg.

This type of distortion was first observed experimentally by Zamir and Young[8] for nominally zero pressure gradient, incompressible flow along a 90-deg corner formed by two intersecting plates with an airfoil-shaped leading edge. In a subsequent study, El-Gamal and Barclay[9] utilized the same corner configuration, but with a sharp (6-deg wedge) leading edge, and did not observe distorted isovel contours in the corner region. On the basis of these results, the authors attributed the appearance of contour distortion in Zamir and Young's experiments to the airfoil-shaped leading edge used in that study.

In a subsequent paper, Zamir and Young[10] agree with this point of view, but argue that the distortion-free results observed by El-Gamal and Barclay[9] were influenced by favorable pressure gradient effects that had a stabilizing in-

Received June 25, 1985; revision received Jan. 13, 1986.

*Graduate Research Assistant, Department of Mechanical Engineering.

†Professor, Department of Mechanical Engineering. Member AIAA.

‡Research Scientist, Applied Computational Aerodynamics Branch. Member AIAA.

fluence on the flow. On the basis of their most recent experiments, Zamir and Young[10] contend that isovel distortion, when it appears in zero pressure gradient corner flow, is due primarily to local flow separation and/or transition near the sharp leading edge, rather than to the intrinsic effect of the corner on the flow. The present numerical study was undertaken in order to determine whether this type of distortion exists in supersonic laminar square duct flow, a confined corner flow that develops in the presence of a mild adverse pressure gradient.

Conservation Equations and Numerical Method

The time-dependent form of the conservation equations for mass, momentum, and energy were analyzed in conjunction with an equation of state based on assumed ideal gas behavior. These equations were written in strong conservation form after the thin-layer approximations had been applied, and were solved on the CRAY X-MP computer by means of a modified form of the ARC-3D code developed by Pulliam and Steger.[11] The computational grid in the cross plane consisted of a 31×31 mesh grid over a quadrant of the flow. A hyperbolic tangent function was used to cluster nodes near the walls; the streamwise nodes were all equally spaced at $\Delta x_1/D = 0.435$, where x_1 is the streamwise coordinate measured from the duct inlet and D is the duct width.

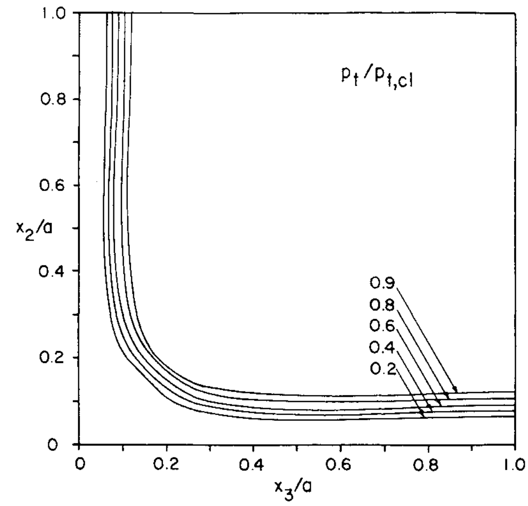
The computations were performed over a development length $0 \leq x_1/D \leq 20$ for an assumed spanwise uniform flow at the duct inlet, with a prescribed inlet Mach number of 4.0 and a unit Reynolds number based on duct width of 1.8×10^6/m. The exiting flowfield was fixed by specifying conditions in the last streamwise computational plane (located one mesh width downstream of the duct exit plane) as a linear extrapolation of calculated flow conditions at $x_1/D = 20$. Additional details of the computational procedure and specified boundary conditions are given by Davis et al.[12]

Results and Discussion

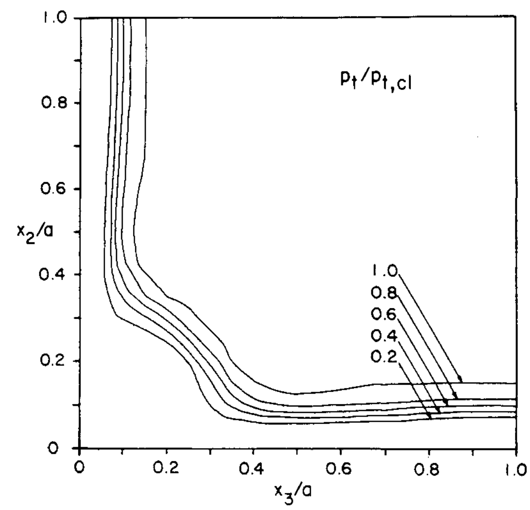
Total pressure contours and crossflow vectors predicted in a quadrant of the flow at three streamwise locations are shown in Figs. 1 and 2, respectively. In these figures x_2 and x_3 are wall coordinates measured from the corner (at 0,0), a corresponds to the duct half width, $p_t/p_{t,cl}$ is the local total pressure referred to its centerline value (at 1,1), and $u_r/u_{1,cl}$ is the resultant crossflow velocity (defined as the vector sum u_2 and u_3) relative to the axial velocity component at the centerline.

The results shown in Figs. 1 and 2 indicate that local flow behavior in the cross plane is strongly dependent on streamwise location. At $x_1/D = 5.22$, crossflow in the vicinity of the corner bisector is directed radially outward (Fig. 2a), with no apparent distortion of total pressure contours in this region (Fig. 1a). These results are similar to those based on the unbounded corner flow analyses of Refs. 1–4. A comparison of Figs. 2a and 2b shows that cross flow immediately adjacent to each bounding wall of the corner reverses direction between $x_1/D = 5.22$ and $x_1/D = 10$. Figure 2b indicates that this reversal is associated with the formation of two discrete secondary flow vortices which are centered about the corner bisector. Figure 1b shows that these vortices distort total pressure contours in the vicinity of the corner bisector at $x_1/D = 10$ in a manner similar to that predicted by Nomura[7] for zero pressure gradient, unbounded corner flow. Close observation of Fig. 2b also reveals that although crossflow along the corner bisector at $x_1/D = 10$ is basically outward, crossflow in the immediate vicinity of the corner is directed toward the corner. These results are qualitatively the same as those predicted by Ghia[2] in his analysis of unbounded corner flow.

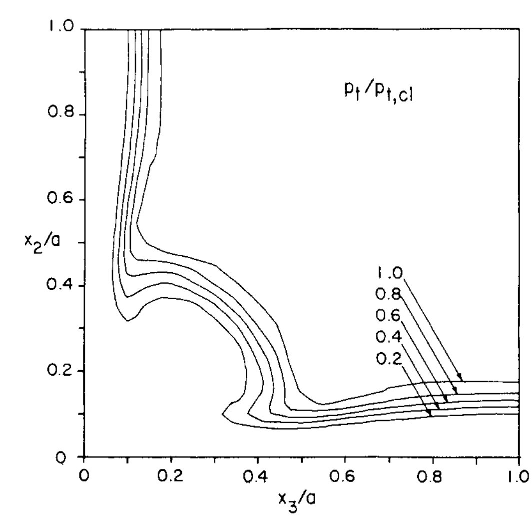
As the flow continues to develop beyond $x_1/D = 10$, the present numerical results show that the flow separates locally in the corner region near $x_1/D = 17$, and that a locally reversed



a) $x_1/D = 5.22$.

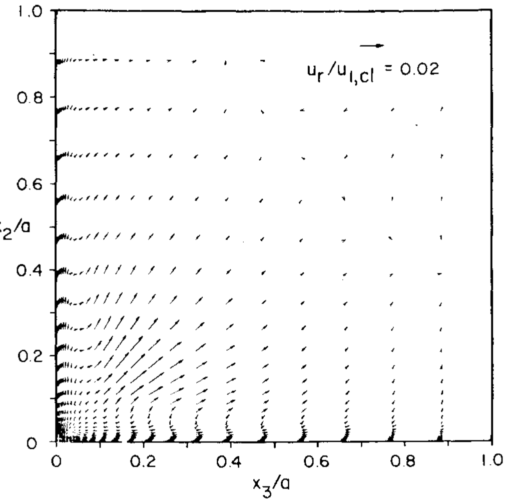


b) $x_1/D = 10$.

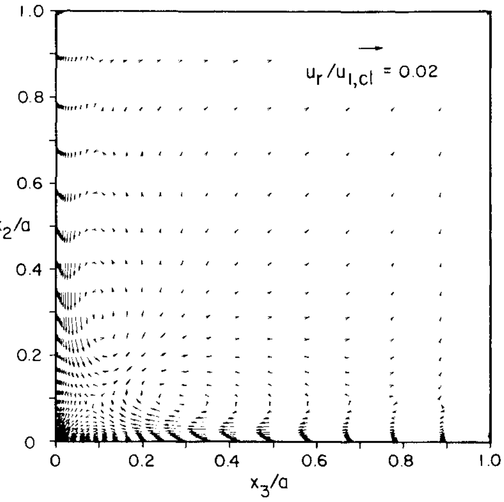


c) $x_1/D = 20$.

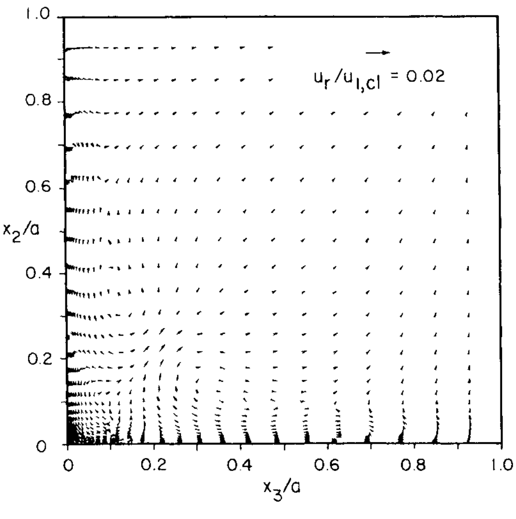
Fig. 1 Predicted total pressure contours for laminar flow.



a) $x_1/D = 5.22$.



b) $x_1/D = 10$.



c) $x_1/D = 20$.

Fig. 2 Predicted crossflow vectors for laminar flow.

axial flow exists along the corner bisector interval $0 < x_2'/a' \leq 0.05$ at $x_1/D = 20$, where x_2' is a coordinate measured from the corner along the corner bisector and a' is the diagonal half width. There is a corresponding increase in total pressure contour distortion between $x_1/D = 10$ and $x_1/D = 20$ (compare Figs. 1b and 1c), even though the strength of the secondary flow has diminished in the near vicinity of the corner (compare Figs. 2b and 2c). This behavior can possibly be explained by noting that a weaker secondary flow at $x_1/D = 20$ may have a stronger convecting influence on the primary flow when primary flow momentum and energy are reduced in the near corner region by virtue of separation bubble formation.

Conclusions

Supersonic laminar flow development in a constant-area square duct exhibits some characteristics which are similar to those observed for unbounded corner flow, yet others which are distinctly different. One of the distinguishing features of supersonic laminar square duct flow is the formation of two contrarotating secondary flow vortices centered about the corner bisector, a phenomenon which apparently does not occur in unbounded corner flow. This secondary flow causes an outward bulging of total pressure contours in the vicinity of the corner bisector for wholly attached flow conditions. When the flow separates locally in the corner region further downstream, total pressure contours become more distorted in the presence of a diminished secondary flow.

Acknowledgment

Funds for the support of this study have been allocated by the NASA Ames Research Center, Moffett Field, California, under Interchange NCA2-1R850-401.

References

[1]Rubin, S. G. and Grossman, B., "Viscous Flow Along a Corner: Numerical Solution of the Corner Layer Equations," *Quarterly of Applied Mathematics,* Vol. 29, July 1971, pp. 169–186.

[2]Ghia, K. N., "Incompressible Streamwise Flow Along an Unbounded Corner," *AIAA Journal,* Vol. 13, July 1975, pp. 902–907.

[3]Barclay, W. H. and Ridha, A. H., "Flow in Streamwise Corners of Arbitrary Angle," *AIAA Journal,* Vol. 18, Dec. 1980, pp. 1413–1420.

[4]Weinberg, B. C. and Rubin, S. G., "Compressible Corner Flow," *Journal of Fluid Mechanics,* Vol. 56, 1972, pp. 753–774.

[5]Ghia, K. N. and Davis, R. T., "A Study of Compressible Potential and Asymptotic Viscous Flows for Corner Region," *AIAA Journal,* Vol. 12, March 1974, pp. 355–359.

[6]Mikhail, A. G. and Ghia, K. N., "Viscous Compressible Flow in the Boundary Region of an Axial Corner," *AIAA Journal,* Vol. 16, Sept. 1978, pp. 931–939.

[7]Nomura, Y., "Analysis of a Characteristic of Laminar Corner Flow," *AIAA Journal,* Vol. 20, July 1982, pp. 1020–1022.

[8]Zamir, M., and Young, A. D., "Experimental Investigation of the Boundary Layer in a Streamwise Corner," *The Aeronautical Quarterly,* Vol. 21, Nov. 1970, pp. 313–339.

[9]El-Gamal, H. A. and Barclay, W. H., "Experiments on the Laminar Flow in a Rectangular Streamwise Corner," *The Aeronautical Quarterly,* Vol. 29, May 1978, pp. 75–97.

[10]Zamir, M. and Young, A. D., "Pressure Gradient and Leading Edge Effects on the Corner Boundary Layer," *The Aeronautical Quarterly,* Vol. 30, Aug. 1979, pp. 471–487; also Vol. 30, Nov. 1979, p. 606.

[11]Pulliam, T. H. and Steger, J. L., "Implicit Finite-Difference Simulations of Three-Dimensional Compressible Flow," *AIAA Journal,* Vol. 18, Feb. 1980, pp. 159–167.

[12]Davis, D. O., Gessner, F. B., and Kerlick, G. D., "Experimental and Numerical Investigation of Supersonic Turbulent Flow Through a Square Duct," *AIAA Journal,* Vol. 24, Sept. 1986, pp. 1508–1515.